$$\frac{n^2 - 3n - 70}{8n + 56} \div \frac{10 + 9n - n^2}{n^2 - 7n - 8}$$

Inverse Trig Funcons

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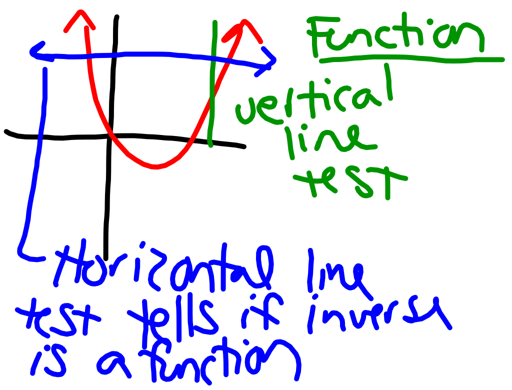
Inverse Funcons

- The domain of the inverse is the range of the original funcon
- The range of the inverse is the domain of the original funcon
- Switching x and y



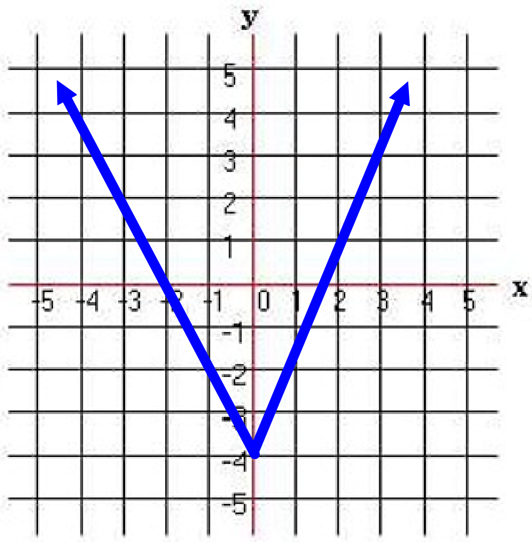
To determine whether the inverse is a funcon…

- Switch x and y values and determine whether the domain of inverse is paired with only one value in the range (domain can not repeat)

Determine the inverse of the relation and decide whether it is a function
Relation: (2,3) (4,2) (5,2) (-3,3) (3,3)
Inverse:

Determining whether the inverse is a funcon from the graph of the original

- You do the vercal line test to determine whether the graph represents a funcon
- To determine whether its inverse would be a funcon, then you would do a horizontal line test



Finding the Inverse of a Relaon

What is the inverse of relaon s?

Relaon s

X	Y
0	-1
2	0
3	2
4	3

x	$y=f^{-1}(x)$
-1	0
0	2
2	3
3	4

function
x can't be repeated

To find the inverse of a funcon

- Notaon: f $f^{-1}(x)=$
- Steps:
- Replace f(x) with y
- Switch the x and y variable
- Solve for y
- Replace y with $f^{-1}(x)=$

$f(x)=2x^2-3$

$y=2x^2-3$

$x=2y^2-3$

$+3 \quad +3$

$\frac{x+3}{2}=\frac{2y^2}{2}$

$\frac{x+3}{2}=y^2$

$\pm\sqrt{\frac{x+3}{2}}=y$

$f^{-1}(x)=\pm\sqrt{\frac{x+3}{2}}$

Find the following

$f(x) = 2x^2 - 3$

$g(x) = 5x - 2$

$h(x) = \sqrt{x} + 1$

$m(x) = \dfrac{2x}{3}$

$f^{-1}(x) =$

$g^{-1}(x) =$

$h^{-1}(x) =$

$m^{-1}(x) =$

$f^{-1}(2) =$

Find the following

$f(x) = 4x^2 - 1$

$g(x) = -x - 1$

$h(x) = \sqrt{x} + 2$

$m(x) = \dfrac{x}{5}$

$f^{-1}(x) =$

$g^{-1}(x) =$

$h^{-1}(x) =$

$m^{-1}(x) =$

$f^{-1}(-3) =$

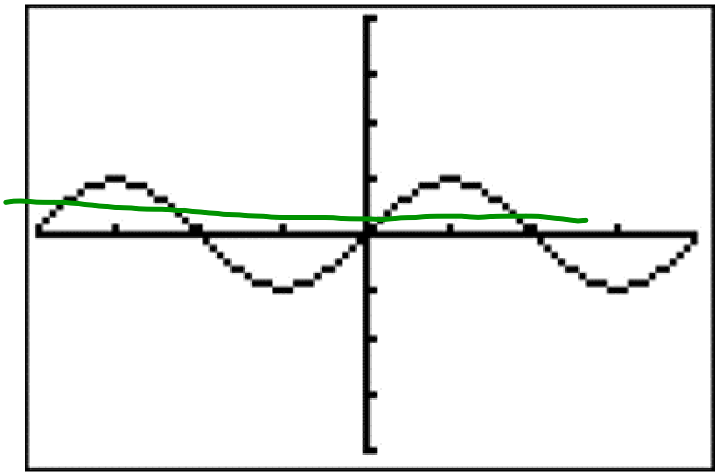
Things to Remember!

- If no horizontal line intersects the graph of a funcon more than once, the funcon is one-to-one and has an inverse funcon.

- If the point (a,b) is on the graph of , then the point (b,a) is

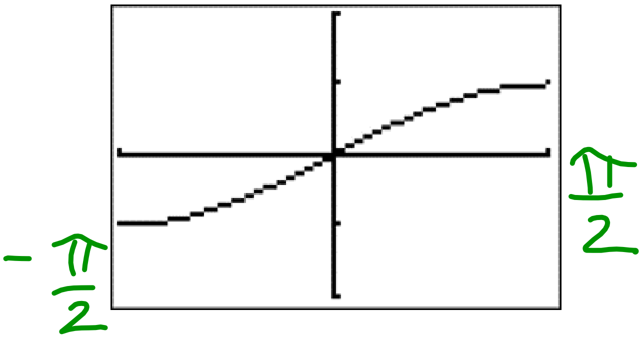
  on the graph of the inverse funcon denoted $f^{-1}(x) =$

Inverse Sine Funcon



The horizontal line test shows that the sine funcon is not one-to-one and has no inverse funcon.

The Restricted Domain



If we take the poron of the sine curve between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$,

$[-\frac{\pi}{2}, \frac{\pi}{2}]$

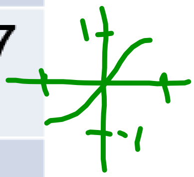
the restricted funcon will pass the horizontal line test and we can talk about its inverse.

Graph the inverse sine

$y = \sin x$

$(-\infty, \infty)$ $[-1, 1]$

x	y
$\frac{-\pi}{2}$	-1
$\frac{-\pi}{4}$	-.707
0	0
$\frac{\pi}{4}$	.707
$\frac{\pi}{2}$	1



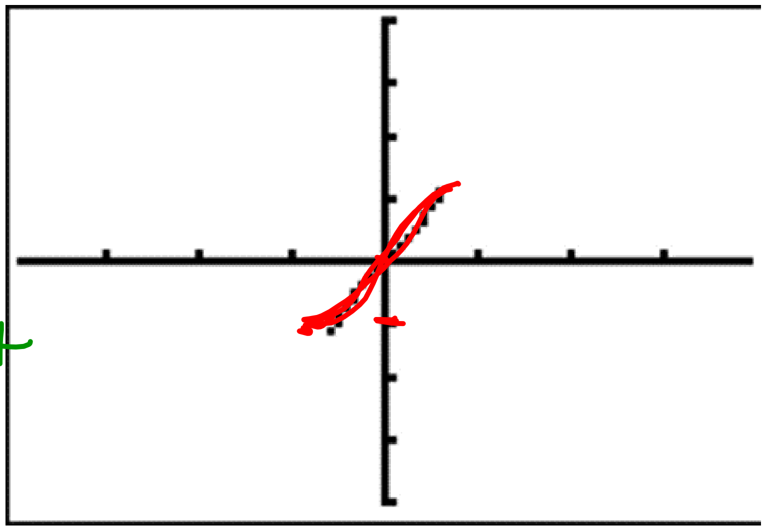
$y = \sin^{-1} x$

$[-1, 1]$ $[-\frac{\pi}{2}, \frac{\pi}{2}]$

x	y
-1	$\frac{-\pi}{2}$
-.707	$\frac{-\pi}{4}$
0	0
.707	$\frac{\pi}{4}$
1	$\frac{\pi}{2}$

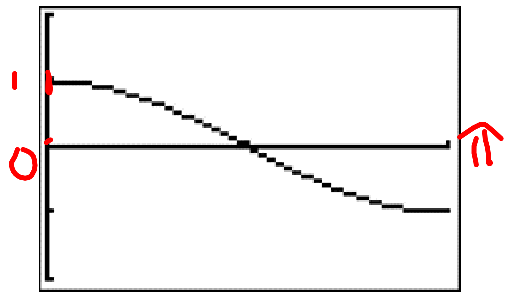
Graph of Inverse Sine

x	y
-1	$\frac{-\pi}{2}$ ≈ -1.57
-.707	$\frac{-\pi}{4}$
0	0
.707	$\frac{\pi}{4}$
1	$\frac{\pi}{2}$



Inverse Cosine Funcon

- As with the sine, the domain must again be restricted. For cosine we restrict to $[0, \pi]$



Graph the Inverse Cosine

$y = \cos x$

x	y
0	1
$\frac{\pi}{4}$	.707
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	-.707
π	-1

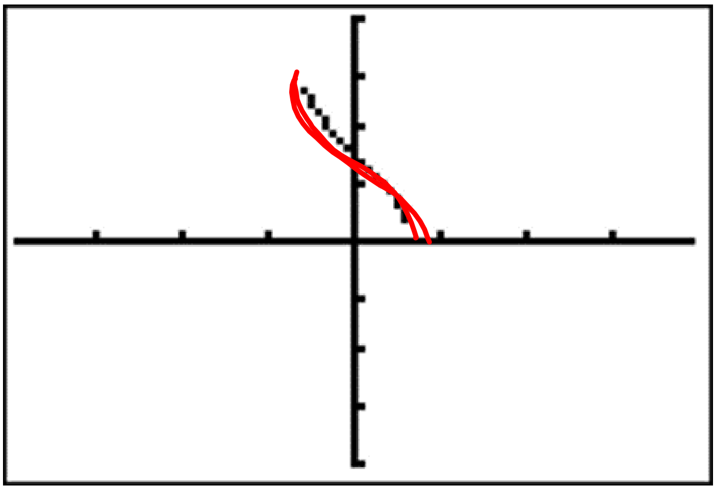
$y = cos^{-1}x$

x	y
1	0
.707	$\frac{\pi}{4}$
0	$\frac{\pi}{2}$
-.707	$\frac{3\pi}{4}$
-1	π

Graph of Inverse Cosine

$y = cos^{-1}x$

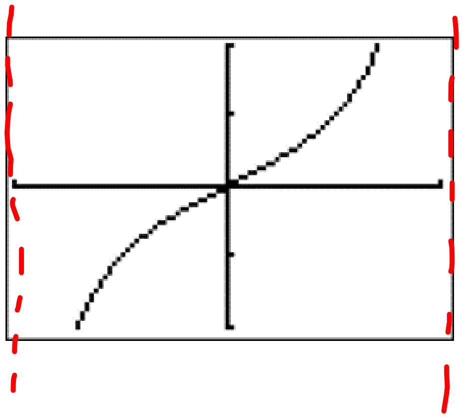
x	y
1	0
.707	$\frac{\pi}{4}$
0	$\frac{\pi}{2}$
-.707	$\frac{3\pi}{4}$
-1	π



Inverse Tangent Funcon

The domain of the tangent must be restricted as the sine to

$$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$



Graph the Inverse Tangent

$y = \tan x$

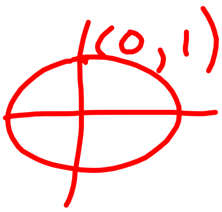
x	y
$\frac{-\pi}{2}$	und
$\frac{-\pi}{4}$	-1
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	und

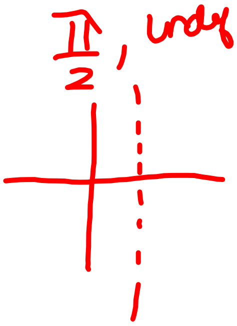
$y = tan^{-1}x$

x	y
und	$\frac{-\pi}{2}$
-1	$\frac{-\pi}{4}$
0	0
1	$\frac{\pi}{4}$
und	$\frac{\pi}{2}$

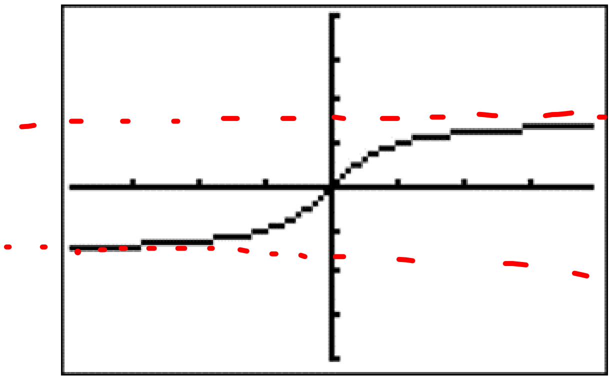
Graph of Inverse Tangent

$$y = tan^{-1}x$$



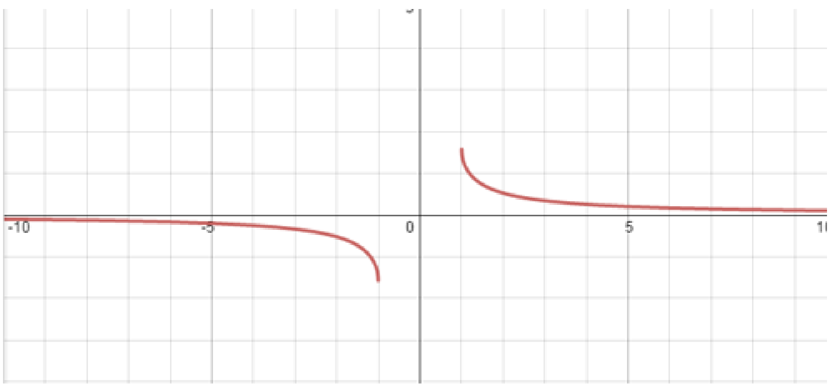


x	y
und	$\frac{-\pi}{2}$
-1	$\frac{-\pi}{4}$
0	0
1	$\frac{\pi}{4}$
und	$\frac{\pi}{2}$



Graph of inverse cosecant

- Domain of sine again restricted to $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ $\csc^{-1}$

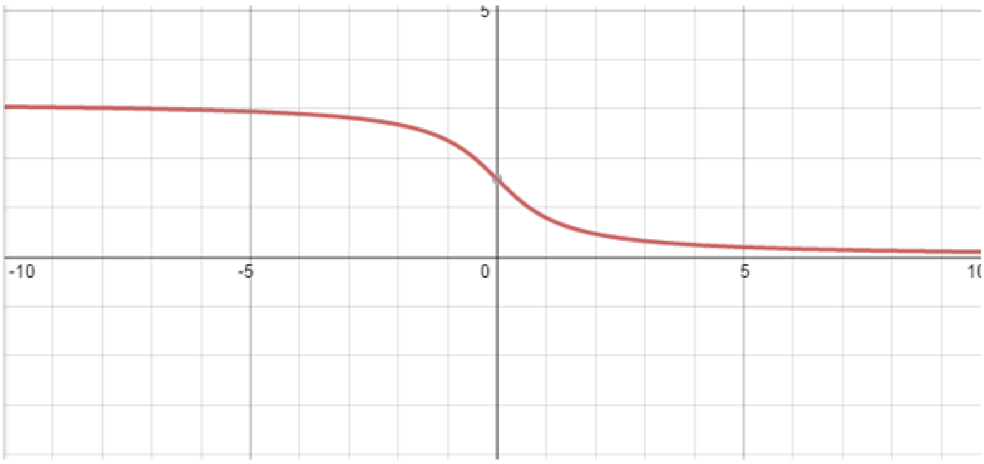


Graph of Inverse Secant

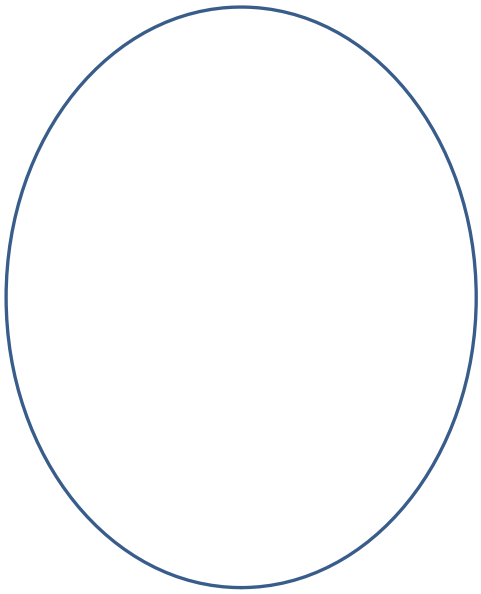
Domain of cosine again restricted to [0,]

Graph of Inverse Cotangent

Domain of tangent again restricted to [0,]



Defined Domains



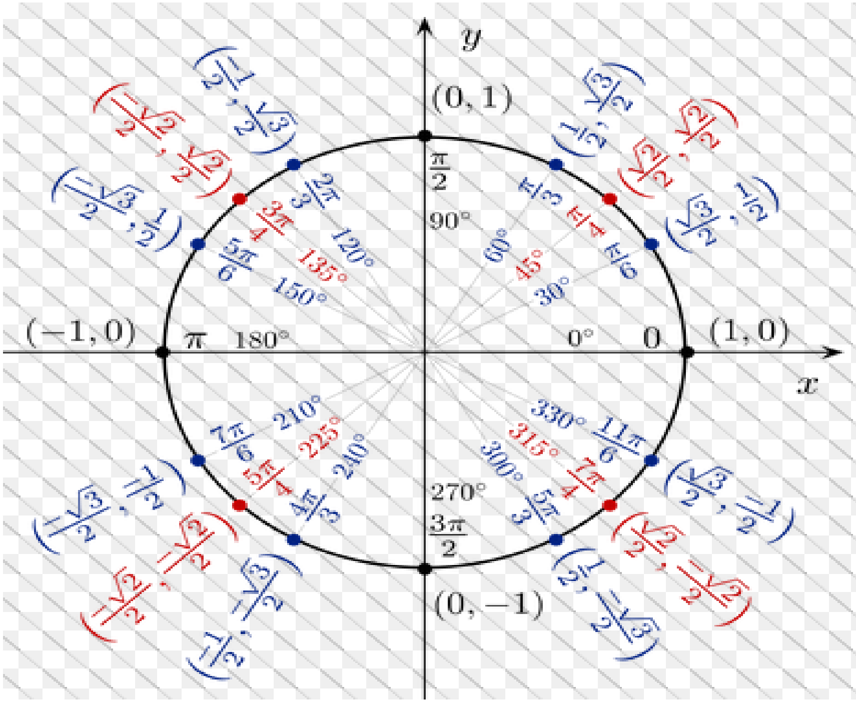
Evaluang Inverses

Consider $\sin\frac{\pi}{6} = \frac{1}{2}$.

When we put in an **angle**, the result is a **ratio**.

When evaluating an inverse, we put in a **ratio** and the result is an **angle**.

$$sin^{-1}\ \frac{1}{2} = \frac{\pi}{6}$$

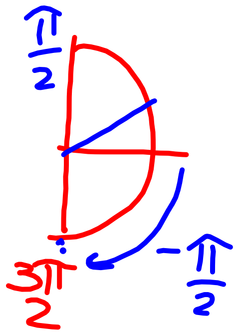

y
(0, 1)
$\frac{\pi}{2}$
90°
(−1, 0)
π
180°
0°
0
(1, 0)
x
270°
$\frac{3\pi}{2}$
(0, −1)
30°
45°
60°
120°
135°
150°
210°
225°
240°
300°
315°
330°

Evaluate $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

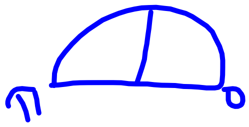
$$sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

or

$$\arcsin\left(\frac{\sqrt{2}}{2}\right)$$



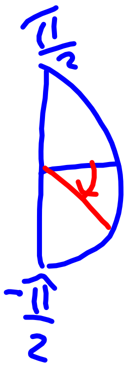
Evaluate



$$cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Evaluate

$tan^{-1}(-1) = -\frac{\pi}{4}$



Evaluate

$$sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$$

Evaluate

$$sec^{-1} 2 =$$

Evaluate

$$tan^{-1}\left(\tan\frac{\pi}{4}\right) =$$

$\text{Tan}^{-1}(1) =$

$\uparrow$

$\frac{\pi}{4}$

Evaluate

$sin^{-1}(\sin \pi) =$

Evaluate

$[0, \pi]$

$$\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right) =$$

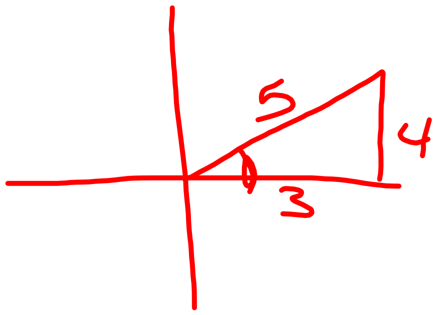
$\sin\left(\frac{\pi}{4}\right)$

$\frac{\sqrt{2}}{2}$

Evaluate

$$\cos\left(sin^{-1}\left(\frac{4}{5}\right)\right) =$$

$\frac{3}{5}$



Evaluate

$$\cot\left(tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) =$$